

5 Impact Minimization Using the Center of Percussion

5.1 Introduction

This chapter establishes the theoretical basis of Center of Percussion (CoP) for the planar and the three-dimensional case, where also the notion of Coefficient of Impact Design is presented. The effect of uncertainties on system or impact parameters are examined using non-dimensional variables. This way it is possible to assess the performance of a robotic operation which incorporates impacts, and estimate the magnitude of the undesired effects which may lead a task unsuccessful. Next, the theory of CoP is established for the multibody systems using the Newton-Euler Approach (NEA). Its use in robotic systems is presented and a control method to exploit the phenomenon at non-free joints is shown. For this reason a controller which exploits the CoP is proposed. In the literature CoP has been used mainly as a reference point; in fact some other kinds of controllers (e.g. PD) was used to control the motor torques, [2] and [4]. To the best of author’s knowledge this is the first time that the CoP is used in the core calculations of the control torques. Implementation guidelines for various manipulator types are discussed. Finally, simulations of a planar space robot system, and a space robot with a 3R manipulator confirm the benefit of using the CoP during tasks that include impacts. The initial concept has been described in [121] and [125], and necessary theoretical and analytical proofs will be included here for convenience. The extension of the theory as well as the controller have been presented in [127] and [128].

5.2 Generalized Theory of Center of Percussion

The CoP is a property of bodies which are able to rotate about a fixed axis. If an impact occurs at the CoP, the reaction force which is exerted on the fixed rotation axis (i.e. on the bearings of the rotational joint), tends to a minimum including zero. More specifically, let a beam, see Figure 5-1, that can rotate about a Rotation Axis (RA). An impact occurs at a point on the longitudinal axis and the overall movement of the body consists of the superposition of:

(i) a translation in the direction of the impulse, therefore an inertial force is exerted at its RA and

(ii) a rotation around its Center of Mass (CoM), thus exerting an inertial force at the RA, where in this case its magnitude is related to the relative position of the impact point with respect to the CoM, whereas its direction is opposite to the reaction force due to translation. If the impact occurs at the CoP, the magnitude of the latter reaction force is equal to the magnitude of the reaction force exerted due to the translation. Therefore in this case the vectorial sum of the forces exerted on the RA is zero – at least in principle.
5.3 Center of Percussion for 2-D systems

To study the CoP concept analytically, consider the free body diagram in Figure 5-2. Assume an impact force at some point (Impact Point - IP) along the longitudinal axis. For purposes of generality, the impact force can have any direction. A balance of forces and moments yields,

\[-m \cdot \dot{v}_{cm} \cdot \sin \theta = -m \cdot \ddot{\theta} \cdot r_{cm} \cdot \sin \theta = N_x - F_{imp} \cdot \cos(\phi + \theta)\]  

\[m \cdot \dot{v}_{cm} \cdot \cos \theta = m \cdot \ddot{\theta} \cdot r_{cm} \cdot \cos \theta = N_y - F_{imp} \cdot \sin(\phi + \theta)\]  

\[I^r \cdot \ddot{\theta} = -l \cdot F_{imp} \cdot \sin \phi\]

Figure 5-2. Free body diagram of a beam under impact force

where all symbols are defined in Figure 5-2 and \(m\) is the mass of the body. The body polar inertia \(I^r\) around the RA at O is,

\[I^r = I^c + m \cdot r_{cm}^2\]  

where \(I^c\) is the body polar inertia around the CoM. The velocity \(v_{cm}\) of the CoM is given by:
\[ v_{cm} = \dot{\theta} \cdot r_{cm} \] (116)

The impact point is located at distance

\[ l = l_{cop} + r = \left( r_{cop} + r_{cm} \right) + r \] (117)

from point O, where \( r \) is the deviation of IP from the CoP (negative if the deviation is towards the CoM and positive otherwise). Integrating (112) - (114) for infinitesimal time, the forces are transformed into corresponding impulses and the accelerations into corresponding velocities, e.g. for the reaction force \( N_s \), the corresponding impulse \( \Omega_{Ns} \) is

\[ \Omega_{Ns} = \lim_{\varepsilon \to 0} \int_0^\varepsilon N_s \, dt \] (118)

where \( \varepsilon \) is the duration of the impact. Other variables, including the rotation angle \( \theta \) and the impact angle \( \phi \), remain unchanged due to the quasi-static assumption. Therefore (112) - (114) become

\[ m \cdot v_{cm} \cdot \sin \theta = \Omega_{N_y} + \Omega_{F_{sw}} \cdot \cos(\phi + \theta) \] (119)

\[ m \cdot v_{cm} \cdot \cos \theta = \Omega_{N_x} - \Omega_{F_{sw}} \cdot \sin(\phi + \theta) \] (120)

\[ I_o \cdot \dot{\theta} = \Omega_{F_{sw}} \cdot l \cdot \sin \theta \cdot \cos(\phi + \theta) - \Omega_{F_{sw}} \cdot l \cdot \cos \theta \cdot \sin(\phi + \theta) \Rightarrow \]

\[ I_o \cdot \dot{\theta} = \Omega_{F_{sw}} \cdot l \left[ \sin \theta \cdot \cos(\phi + \theta) - \cos \theta \cdot \sin(\phi + \theta) \right] \Rightarrow \]

\[ I_o \cdot \dot{\theta} = -\Omega_{F_{sw}} \cdot l \cdot \sin \phi \] (121)

As \( \dot{\theta} \) is the angular velocity, from (116),

\[ I_o \cdot \dot{\theta} = I_o \cdot \frac{v_{cm}}{v_{cm}} \Rightarrow v_{cm} = -\frac{\Omega_{F_{sw}} \cdot l \cdot r_{cm}}{I_o} \cdot \sin \phi \] (122)

and replacing (122) in (120) and (121)

\[ \frac{\Omega_{Ns} - \Omega_{F_{sw}} \cdot \sin(\phi + \theta)}{m \cdot \cos \theta} = -\frac{\Omega_{F_{sw}} \cdot l \cdot r_{cm}}{I_o} \cdot \sin \phi \Rightarrow \]

\[ \Rightarrow \Omega_{Ns} = \Omega_{F_{sw}} \cdot \sin(\phi + \theta) - \frac{\Omega_{F_{sw}} \cdot l \cdot r_{cm} \cdot m}{I_o} \cdot \cos \theta \cdot \sin \phi \] (123)

\[ \Rightarrow \Omega_{Ns} = \Omega_{F_{sw}} \cdot \left( \sin(\phi + \theta) - \frac{l \cdot r_{cm} \cdot m}{I_o} \cdot \cos \theta \cdot \sin \phi \right) \]
\[
\Omega_{Ny} + \Omega_{r_m} \cdot \cos(\phi + \theta) \cdot \cos \theta = -\frac{\Omega_{r_m} \cdot l \cdot r_m \cdot m}{I_o} \cdot \sin \phi \Rightarrow \\
\Rightarrow \Omega_{Ny} = -\Omega_{r_m} \cdot \cos(\phi + \theta) \cdot \frac{\Omega_{r_m} \cdot l \cdot r_m \cdot m}{I_o} \cdot \sin \theta \cdot \sin \phi \Rightarrow \\
\Rightarrow \Omega_{Ny} = \Omega_{r_m} \cdot \left(-\cos(\phi + \theta) - \frac{l \cdot r_m \cdot m}{I_o} \cdot \sin \theta \cdot \sin \phi \right)
\]

The magnitude of the impulse of the reaction force \( N, \Omega_N \), is given by

\[
\Omega_N^2 = \Omega_{Ny}^2 + \Omega_{Nm}^2 = \Omega_{r_m}^2 \left[1 + \left(C_{ID}^2 - 2 \cdot C_{ID} \right) \sin^2 \phi \right]
\]

with

\[
C_{ID} = \frac{l \cdot r_m \cdot m}{I_o}
\]

where \( C_{ID} \) is the Coefficient of Impact Design, a term which relates the physical characteristics of the body to the location of the impact. Equations are simplified using the non-dimensional impulse

\[
\tilde{\Omega}_N = \sqrt{\frac{\Omega_N^2}{\Omega_{r_m}^2}}
\]

Equation (125) does not depend on the beam angle \( \theta \), but depends on the angle of impact \( \phi \) and the impact point and body parameters due to (126). To find the CoP, the reaction impulse is set to zero, yielding

\[
\tilde{\Omega}_N = 0 \iff \phi = \pm \sin^{-1} \left[\left(2 \cdot C_{ID} - C_{ID}^2 \right)^{1/2} \right] = \pm \sin^{-1} \beta
\]

The term in the brackets is valid for \( \beta \leq 1 \), while the radicand of the square root has real values only for \( 0 < C_{ID} < 2 \), and thus \( \beta \geq 1 \). Therefore, the reaction impulse will be eliminated iff

\[
\tilde{\Omega}_N = 0 \iff \beta = 1 \iff C_{ID} = 1 \quad \text{and} \quad \phi = \pm \pi/2
\]

The sign is related to the force direction. Equation (129) shows the uniqueness of the CoP along the longitudinal axis of a beam for a rigid body. From (126), and using (115), (129) and (117) for \( r = 0 \) (impact occurs at the CoP), it can be found that the CoP’s location is

\[
C_{ID} = \frac{l \cdot r_m \cdot m}{I_o} = 1 \iff r_{COP} = \frac{I^c}{r_{cm} \cdot m}
\]

Equation (130) is the equation that locates the CoP of a rigid body in 2-D. Interestingly if \( r_{cm} \to 0 \iff r_{COP} \to +\infty \), i.e. the reaction forces cannot be eliminated on a statically balanced body.
On the other hand if \( r_{cm} \to +\infty \Leftrightarrow r_{cop} \to 0 \), i.e. on a body with all its mass concentrated at a point away from the RA, the CoP is at the same point. Note also from (129) and (125), that when \( \phi \neq \pm \pi/2 \), then the non-dimensional impulse cannot be eliminated fully; this is due to the fact that in such a case there is always an impact force component parallel to the longitudinal axis, whose line of action passes through O, and therefore it does not produce any moment about O. Thus it acts directly on the joint bearings, without leaving any margin to cancel it out.

### 5.4 Center of Percussion for 3-D systems

Consider the rigid body in Figure 5-3 whose CR is located at the spherical joint O and a force \( F_{imp} \) acting on it at an Impact Point (IP).

![Figure 5-3. A 3-D rigid body rotating around a spherical joint and the vectors used to derive the CoP conditions in the 3-D case.](image)

The equations of motion for the CS \( a: \{xyz\} \) are

\[
\sum a F_o = m \cdot a \ddot{v}_{cm} = a N + a F_{imp} \tag{131}
\]

\[
\sum a M_o = d/dt \left( a I^* \cdot a \omega \right) = \left( a l_{imp} \times a F_{imp} \right) \tag{132}
\]

where \( N \) is the reaction force at O, \( I^* \) is the inertia matrix of the body with respect to O, \( F_o \) and \( M_o \) is the vectorial sum of forces and moments with respect to O correspondingly, \( v_{cm} \) is the linear velocity of the COM and \( \omega \) is the angular velocity of the body around O. For any CS, the following holds (\( I_3 \) is the 3x3 unit matrix)

\[
v_{cm} = \omega \times r_{cm} \tag{133}
\]

\[
l_{imp} = r_{cm} + r_{imp} = r_{cm} + r_{cop} + r = l_{cop} + r \tag{134}
\]
\[
I' = I + m \left( r_{cm} \cdot r_{cm} \cdot 1_3 - r_{cm} \cdot r_{cm}^T \right)
\] (135)

where \(I\) is the inertia matrix with respect to COM, and the vectors are defined in Figure 5-3. Integrating for a short duration (like in (118)) (131) and (132), and using (133)

\[
m \cdot \dot{v}_{cm} = m \left( \omega \times r_{cm} \right) = \Omega_N + \Omega_{f_{eq}}
\] (136)

\[
^aI' \cdot ^a\omega = ^aI_{imp} \times \Omega_{f_{eq}}
\] (137)

and performing some algebraic manipulation of (136) and (137) results in

\[
^aI' \cdot ^a\omega = \left[ ^aI_{imp} \times \left( m \cdot ^a\omega \times ^a\omega_{cm} - ^a\Omega_N \right) \right]
\] (138)

where \(^a\Omega_N\) is the reaction impulse at point O.

The impact force \(F_{imp}\) and the corresponding reaction force \(N\) can be considered as the vectorial sum of normal components \(F_{imp\perp}\) and \(N_{\perp}\), and parallel components \(F_{imp\parallel}\) and \(N_{\parallel}\) to the impact vector \(I_{imp}\), irrespective of the CS. The same stands for their corresponding impulses therefore

\[
\Omega_{f_{eq}} = \Omega_{f_{eq}\perp} + \Omega_{f_{eq\parallel}}
\] (139)

\[
\Omega_N = \Omega_{N\perp} + \Omega_{N\parallel}
\] (140)

The parallel component \(\Omega_{N\parallel}\), which corresponds to the reaction of the parallel component \(\Omega_{f_{eq\parallel}}\) can not be eliminated. This is because \(F_{imp\parallel}\) does not produce a moment around O and its line of action passes through the CR. Therefore, the focus is to eliminate the \(\Omega_{N\perp}\) due to \(F_{imp\perp}\). This requirement can be summarized as

\[
I_{imp} \cdot F_{imp\perp} = 0
\] (141)

Note that (141) is valid for any CS. This condition is qualitatively similar to the angle requirement in (129). Using (140) in (138), and given that the cross product of parallel vectors is zero, one finds that,

\[
^aI' \cdot ^a\omega - ^aI_{imp} \times \left( m \cdot ^a\omega \times ^a\omega_{cm} \right) = ^a\Omega_{N\perp} \times ^aI_{imp}
\] (142)

To zero \(\Omega_{N\perp}\), it is required that the left side of (142) must be equal to zero (\(^aI_{imp} \neq 0\) otherwise the impact occurs at the spherical joint). According to the definition of the CoP, for such a point, \(^aI_{imp} = ^aI_{cop}\), and therefore
During impact, the moment with respect to CS \( a : \{xyz\} \) due to \( \mathbf{F}_{\text{imp}} \) is given by

\[
{^a}\mathbf{M} = \mathbf{I}_{\text{cop}} \times {^a}\mathbf{F}_{\text{imp}}
\]  

(144)

Thus the instantaneous axis of rotation is given by

\[
\hat{\mathbf{t}} = \frac{\mathbf{M}}{\|\mathbf{M}\|}
\]  

(145)

Let also unit vectors \( \mathbf{\hat{n}} \) and \( \mathbf{\hat{s}} \) normal to each other and to \( \hat{\mathbf{t}} \) so that an orthogonal CS \( b : \{nst\} \) is formed (the \( \mathbf{\hat{n}} \) or the \( \mathbf{\hat{s}} \) correspond to any arbitrary direction on the \( \{ns\} \) plane as long as they are perpendicular to each other and to \( \hat{\mathbf{t}} \)). The instantaneous angular velocity in this CS is then

\[
{^b}\mathbf{\omega} = \begin{pmatrix} 0 & 0 & \omega_z \end{pmatrix}^T
\]  

(146)

It will be advantageous to write (143) as

\[
{^a}\mathbf{I} \cdot {^a}\mathbf{\omega} = m {^a}\mathbf{r}_{\text{cm}} \times {^a}\mathbf{\omega} \times {^a}\mathbf{l}_{\text{cop}}
\]  

(147)

where \( {^a}\mathbf{r}^\times \) is the matrix that corresponds to a cross product

\[
{^a}\mathbf{r} = \begin{pmatrix} x & y & z \end{pmatrix}^T \Leftrightarrow {^a}\mathbf{r}^\times = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}
\]  

(148)

Note however that the following applies

\[
{^a}\mathbf{I} = {^b}\mathbf{R} \cdot {^b}\mathbf{I} \cdot {^b}\mathbf{R}^T
\]  

(149)

where \( {^a}\mathbf{I} \) is the inertia matrix with respect to the O as seen from CS \( b : \{nst\} \) and \( {^b}\mathbf{R} \) the rotation matrix from CS \( b : \{nst\} \) to CS \( a : \{xyz\} \). The same applies also to \( {^a}\mathbf{r}^\times_{\text{cm}} \) and \( {^a}\mathbf{\omega}^\times \). Eq. (147) can be written as

\[
{^a}\mathbf{I} \cdot {^a}\mathbf{\omega} = m {^a}\mathbf{r}_{\text{cm}} \times {^a}\mathbf{\omega} \times {^a}\mathbf{l}_{\text{cop}} \Rightarrow
\]

\[
= {^b}\mathbf{R} \cdot {^a}\mathbf{I} \cdot {^b}\mathbf{R}^T \cdot {^b}\mathbf{\omega} = m {^b}\mathbf{r}_{\text{cm}} \times {^b}\mathbf{R}^T \cdot {^b}\mathbf{R} \cdot {^b}\mathbf{\omega} \cdot {^b}\mathbf{l}_{\text{cop}} \Rightarrow
\]

\[
= {^b}\mathbf{R} \cdot {^a}\mathbf{I} \cdot {^b}\mathbf{\omega} = m {^b}\mathbf{r}_{\text{cm}} \times {^b}\mathbf{\omega} \times {^b}\mathbf{l}_{\text{cop}} \Rightarrow
\]

\[
= {^b}\mathbf{R} \left( {^a}\mathbf{I} \cdot {^b}\mathbf{\omega} - m {^b}\mathbf{r}_{\text{cm}} \times {^b}\mathbf{\omega} \times {^b}\mathbf{l}_{\text{cop}} \right) = 0
\]  

(150)

However by [139]

\[
\det({^b}\mathbf{R}) = +1
\]  

(151)
thus the only solution for (150) is

$$^b I^o \cdot ^b \omega = m^b r_{cm} \cdot ^b \omega \cdot ^b l_{coup}$$

Equation (152) using (146) and a generic inertia matrix

$$^b I^o = \begin{pmatrix} ^b I_{nn}^o & ^b I_{an}^o & ^b I_{at}^o \\ ^b I_{an}^o & ^b I_{aa}^o & ^b I_{at}^o \\ ^b I_{at}^o & ^b I_{at}^o & ^b I_{tt}^o \end{pmatrix}$$

results in,

$$\omega \cdot \begin{pmatrix} ^b I_{at}^o + m \cdot r_{cm,2} \cdot ^b l_{coup,2} \\ ^b I_{at}^o + m \cdot r_{cm,1} \cdot ^b l_{coup,1} \\ ^b I_{at}^o - m \cdot (r_{cm,1} \cdot ^b l_{coup,1} + r_{cm,2} \cdot ^b l_{coup,2}) \end{pmatrix} = 0$$

where $$r_{cm,i}$$ and $$l_{coup,i}$$ refer to the i^th component of the corresponding vector in CS $$^b \{ns\}$$. For (143) to be valid, the column vector in (154) must be zero. The case where $$\omega_i = 0$$ is trivial (no impact occurs or the body is fixed). Therefore, (154) holds if all the following conditions hold:

i) $$\hat{t}$$ is a principal inertia axis of the body. Then,

$$^b I_{at}^o = ^b I_{tt}^o = 0$$

ii) There is symmetry in mass distribution with respect to the $$\{ns\}$$ plane, i.e.

$$r_{cm,2} = 0$$

iii) Using the last row of (154) and (156), the impact occurs at a point which satisfies

$$^b r_{cm} \cdot ^b l_{coup} = ^b I_{at}^o \cdot m^{-1}$$

By virtue of (134), (135) and (156) the following apply

$$^b r_{cm} \cdot ^b r_{cm} = r_{cm,1}^2 + r_{cm,2}^2$$

$$^b I_{at}^o = ^b I_{at}^o + m \cdot (r_{cm,1}^2 + r_{cm,2}^2)$$

so (157) becomes

$$^b I_{at}^o \cdot m^{-1} = ^b r_{cm} \cdot ^b r_{coup}$$

Equation (160) requires that the IP, the CoM and the CR should be collinear. In the planar case, $$\hat{t}$$ is substituted with $$\hat{z}$$, and (160) reduces to (130). It is reminded that (141) should also apply.
the other hand if only (141) and (160) apply, the reaction forces can be still reduced but not eliminated (note that in this case (160) applies for the projection of the CoM on the \( \{ns\} \) plane). From the analysis of the 3-D case one can deduce that in order to zero the reaction forces on the bearings of the spherical joint, the impact should occur on certain planes, rendering the problem essentially planar. Therefore for the rest of this work it is assumed that the impacts occur on a plane, on which under ideal conditions, the reaction forces can be zeroed.

### 5.5 Robustness to Parametric Uncertainties

#### 5.5.1 Introduction

As it has been already described in Section 5.2, the minimization of the reaction is possible either in the planar case, or at specific planes in the three dimensional case. For this reason, the analysis here will focus to the planar case and necessary adaptations for the 3-D case will be presented later.

It is interesting to examine the effects of changes in the \( C_{ID} \) or in the angle \( \phi \) using the non-dimensional case of (125). Figure 5-4a shows \( \frac{\partial (\hat{\Omega}_n)}{\partial \phi} \) as a function of \( \phi \). It can be seen that the sensitivity is highest when \( C_{ID} = 1 \). Similarly Figure 5-4b shows \( \frac{\partial (\hat{\Omega}_n)}{\partial C_{ID}} \) as a function of \( C_{ID} \). It is observed that the highest sensitivity is at \( \phi = 90^\circ \). This preliminary sensitivity analysis shows that deviations from the normal impact angle yield higher rate of change of the reaction forces than deviations from the impact location of the CoP. Therefore a system which is under impact should try to achieve, prior to the impact, a configuration which allows to accomplish the requirements of the percussion point as in (129) and if this is not achievable it should try to keep the angle of the impact as close to \( \phi = 90^\circ \) as possible. However it is of interest to examine further how deviations of system or impact parameters affect the reaction forces.

#### 5.5.2 Non-dimensional analysis of the uncertainties

To analyze further the effects of the deviation of the IP from the CoP, \( \hat{r} \) is defined as the non-dimensional deviation from CoP, see Figure 5-2

\[
\hat{r} = \frac{r}{r_{coP}} \tag{161}
\]

where \( r \) is the distance of the impact point from CoP. Additionally the notion of the “impact configuration” is defined as “the set of system and impact parameters prior to an impact, necessary to describe the impact behaviour.” In this context the system parameters include the
body parameters of the \( C_{ID} \), and the impact parameters include the impact distance \( \bar{r} \) and impact angle \( \phi \).

![Figure 5-4](a) Impact Angle \( \phi \) in degrees  

![Figure 5-4](b) Coefficient of Impact Design \( C_{ID} \) Change Rate of Non-dimensional Impulse

**Figure 5-4.** Change rate of non-dimensional impulse as (a) Impact angle changes and (b) as the coefficient of impact design changes.

Let now the maximum desired non-dimensional impulse \( \hat{\Omega}_N \) be known. One can define two extremes:

a) The *minimum impact angle*, \( \phi_{\text{min}} \), if and only if the impact occurs at the CoP. More specifically for impact at the CoP (\( C_{ID} = 1 \)), and given \( \hat{\Omega}_N \), one can find from (125):

\[
\left( \hat{\Omega}_N \right)^2 = 1 - \sin^2 \phi_{\text{min}} \Rightarrow \phi_{\text{min}} = \sin^{-1} \left( \left( 1 - \left( \hat{\Omega}_N \right)^2 \right)^{1/2} \right)
\]  

(162)

b) The *maximum distance of the IP from the CoP*, defined non-dimensionally by \( C_{ID,\text{max}} \), if and only if the impact force is normal to the longitudinal axis of the body (as defined by the line which connects the RA and the CoM). For a normal impact (\( \phi = 90^\circ \)) and a given \( \hat{\Omega}_N \), one can find from (125):

\[
C_{ID,\text{max}} = 1 \pm \hat{\Omega}_N
\]  

(163)

Therefore the next question to be answered is for a given \( \hat{\Omega}_N \) what impact configurations are admissible (i.e. impact point distance and impact angle). Note that the impact angle cannot be zeroed, that would mean that the impact is colinear to the longitudinal axis and no reaction reduction measure can be applied on the joint under consideration. Also values of \( C_{ID,\text{max}} > 2 \) or \( C_{ID,\text{max}} < 0 \) are of no interest, as in this case the reaction force due to the developed moment is larger than the impact force, see also (125). In this case the beam becomes a fulcrum.
Using (125) one can find the dependence of $C_{ID}$ to the non-dimensional impulse and the impact angle as,

\[
\left( \tilde{\Omega}_N \right)^2 = 1 + \left[ \left( C_{ID}^2 - 2 \cdot C_{ID} \right) \cdot \sin^2 \phi \right]^{(\phi \neq 90^\circ, \pm 90^\circ)} \Rightarrow \\
\Rightarrow C_{ID}^2 - 2 \cdot C_{ID} - \left( \tilde{\Omega}_N \right)^2 \cdot 1 \cdot \left( \sin^2 \phi \right)^{-1} = 0 \Rightarrow \\
\Rightarrow C_{ID} = 1 \pm \sqrt{1 - \left( \tilde{\Omega}_N \right)^2} \cdot \left( \sin^2 \phi \right)^{-1}
\]

Equation (164) is being plotted for a given $\tilde{\Omega}_N$ in Figure 5-5. Every impact configuration inside the sketched area creates reaction force less than $\tilde{\Omega}_N$. Therefore considering the uncertainties of the body characteristics and the impact point – thus the $C_{ID}$, and the uncertainties on the impact angle estimation – thus the $\phi$, one can estimate how close an impact configuration to the limits of the sketched area is, where the non-dimensional impulse equals $\tilde{\Omega}_N$. This way the impact configuration (including the errors) which is closer to the limits of Figure 5-5, is the extreme case of impact for the system defined.

Another way to examine the relation between $C_{ID}$, $\phi$ and $\tilde{\Omega}_N$ is by using Figure 5-6, where lines of constant impact angles have been drawn. The areas outside the lines of $\phi = 90^\circ$ cannot be achieved.

Figure 5-5. Graph of feasible impacts for a defined $\tilde{\Omega}_N$. 

\[\text{Estimated Nominal Impact Configuration}\\\text{Error Estimation in } C_{ID}\\\text{Error Estimation in impact angle}\\\text{Limit for impact further from the CoM, } +r\\\text{Limit for impact nearer to the CoM, } -r\\\phi_m\rightarrow 90^\circ\]
5.5.3 Assessment of performance during design

Figure 5-6 can be used also for the selection of the body parameters during the design phases of a system or for the performance under known impacts for a known body. More specifically, (126) is rewritten using (166) and (161) thus

$$C_{ID} = \frac{I^c \cdot r_{cm}}{I^c} = 1 + C_{MD} \cdot \bar{r}$$

(165)

where $C_{MD}$ is the Coefficient of Mass Distribution defined as the ratio between the body polar inertia with respect to its CoM, $I^c$, to the body polar inertia with respect to the RA, $I^r$. It is a metric of the “symmetry” of the mass distribution between the CoM and the RA. To clarify this, note that

$$C_{MD} = \frac{I^c}{I^r} = \frac{I^c}{(I^c + m \cdot r_{cm}^2)} \Rightarrow 0 \leq C_{MD} \leq 1$$

(166)

thus it can be seen that

(i) as the CoM is closer to the RA ($r_{cm} \to 0$), there is larger “symmetry” in the mass distribution ($C_{MD} = 1$)

(ii) as the RA is further away from the CoM ($r_{cm} \to +\infty$), this “symmetry” is affected ($C_{MD} \to 0$).
As an example of the use of Figure 5-6, suppose that the control designer sets the following extremes for nominal operation of a manipulator during impacts: \( \phi = 70^\circ \) and \( \tilde{\Omega}_N = 0.4 \). One can find that for the given \( \phi \) and \( \tilde{\Omega}_N \) this corresponds to point B (and C) and from the graph this corresponds to \( C_{ID} = 0.68 \) (and \( C_{ID} = 1.32 \)). Knowing the parameters of the system, thus the \( C_{MD} \), then the maximum deviation of the IP can be found – and eventually the acceptable level of uncertainty during the impact. One can recognise the following areas in Figure 5-6:

(i) BHCEFGB is the area in which any impact is within the requirements,

(ii) BCHB is an area in which the \( \tilde{\Omega}_N \) satisfies the requirement, and at the same time the angle requirement is relaxed, provided however that the IP is nearer to the CoP. Finally,

(iii) ABGA and CDEC are areas that are within the \( \tilde{\Omega}_N \) requirement, but the impact angle must be greater than the minimum requirement. Therefore, in the case the body parameters are known and a controller is implemented to control the body impact configuration prior to impact, using the abovementioned method it is possible to assess its performance and the limits of its application. The more stringent the requirements are with respect to the accepted impulse, the more robust the controller should be.

### 5.5.4 Extensions to the 3-D case

We focus at the plane normal to the instantaneous axis of rotation \( \hat{\mathbf{i}} \) according to CS \( b : \{ n s r \} \). Figure 5-3. Therefore \( C_{ID, \hat{\mathbf{i}}} \) is the Coefficient of Impact Design about \( \hat{\mathbf{i}} \), which relates the physical characteristics of the body to the location of the impact

\[
C_{ID, \hat{\mathbf{i}}} = \frac{\mathbf{b} \cdot \mathbf{r}_{m} \cdot m}{\mathbf{b} \cdot I_{m}^{\hat{\mathbf{i}}}}
\]  

(167)

where \( \mathbf{b} \) is the vector which connects the IP and the RA, \( \mathbf{r}_{m} \) is the vector which connects the CoM and the RA, \( m \) is the mass of the body and \( \mathbf{b} I_{m}^{\hat{\mathbf{i}}} \) the body polar inertia around \( \hat{\mathbf{i}} \).

Additionally the Coefficient of Mass Distribution about \( \hat{\mathbf{i}} \) is

\[
C_{MD, \hat{\mathbf{i}}} = \frac{\mathbf{b} I_{c}^{\hat{\mathbf{i}}}}{\mathbf{b} I_{m}^{\hat{\mathbf{i}}}}
\]  

(168)

which is the ratio between the body polar inertia with respect to its CoM around \( \hat{\mathbf{i}} \), \( \mathbf{b} I_{c}^{\hat{\mathbf{i}}} \), and the body polar inertia with respect to the RA around \( \hat{\mathbf{i}} \), \( \mathbf{b} I_{m}^{\hat{\mathbf{i}}} \). Finally the deviation of the IP from the CoP \( \tilde{n} \) is the non-dimensional deviation from CoP.
\[ \vec{f}_i = \text{sgn}(\vec{b} \cdot \vec{b}_c) \left[ \frac{\vec{b} \cdot \vec{b}_r}{||\vec{b}_c \cdot \vec{b}_r||} \right] \]  

where \( \vec{b} \) is the vector which connects the IP with the CoP. By applying the above definitions, the rest of the analysis is valid for the 3-D case.

### 5.6 CoP on robotic manipulators and impact compensation

#### 5.6.1 Derivation of dynamic equations

To study the application of the CoP concept in robotic systems, it is useful to derive the CoP for multibody systems. This can be achieved using the Newton-Euler Algorithm (NEA), [25].

Consider two adjacent links, **Figure 5-7**.

![Free body diagram of two adjacent manipulator links](image)

**Figure 5-7.** Free body diagram of two adjacent manipulator links.

Note that the external impact force \( \vec{f}_{imp,i} \) acts at the impact point (IP), which is located at \( \vec{r}_{un,i} \) from the link CoM, and \( -\vec{r}_{dist,i} \) from the next joint \( \{i+1\} \). Also note that \( \vec{r}_{cop,i} \) is the vector from the CoM to the CoP and \( \vec{r}_{i} \) is the vector from the CoP to the IP. The rest vectors are defined in **Figure 5-7**. In this work the equations are used in the following form

\[ \dot{\vec{v}}_{c,i} = \frac{d}{dt} \left( \vec{\omega} \times \vec{r}_{c,i} + \vec{v}_i \right) \quad (170) \]

\[ \dot{\vec{F}}_i = m_j \cdot \dot{\vec{v}}_{c,i} \quad (171) \]

\[ \dot{\vec{M}}_j = \frac{d}{dt} \left( \vec{c} \cdot \vec{I} \cdot \vec{\omega} \right) \quad (172) \]

\[ \dot{\vec{f}}_i = \vec{R}_{i+1} \cdot \dot{\vec{f}}_{i+1} - \vec{f}_{imp,i} + \vec{F}_i \quad (173) \]
\[ \dot{n}_i = \dot{\mathbf{M}}_j + \dot{\mathbf{R}}_{ij} \cdot (\dot{\mathbf{n}}_{ij} + \dot{\mathbf{r}}_{cj} \times \dot{\mathbf{F}} + \dot{\mathbf{r}}_{ij} \times \dot{\mathbf{R}}_{ij} \cdot (\dot{\mathbf{t}}_{ij} - \dot{\mathbf{r}}_{imp,j} \times \dot{\mathbf{f}}_{imp,j}) \] (174)

Also

\[ \dot{r}_{imp,j} = \dot{r}_{cj} + \dot{r}_{un,j} \] (175)

\[ \dot{r}_{un,j} = \dot{r}_{cj} + \dot{r}_{ms,j} + \dot{r}_{dist,j} \] (176)

\[ \dot{r}_{ms,j} = \dot{r}_{imp,j} + \dot{r}_i \] (177)

Equations (170) - (174) are integrated for infinitesimal time to yield,

\[
\begin{align*}
\mathbf{C}_i \mathbf{I}_i \cdot \dot{\omega}_i &= \int \left( \dot{\mathbf{n}}_i - \dot{\mathbf{R}}_{ij} \cdot (\dot{\mathbf{n}}_{ij} + \dot{\mathbf{u}}_{ms}) - \dot{\mathbf{P}}_{Cj} \times (\dot{\mathbf{t}}_{ij}) \right) d\tau \\
&= \int \left( \dot{\mathbf{n}}_i - \dot{\mathbf{R}}_{ij} \cdot (\dot{\mathbf{n}}_{ij} + \dot{\mathbf{u}}_{ms}) + \dot{\mathbf{P}}_{Cj} \times (\dot{\mathbf{t}}_{ij}) \right) d\tau \\
&= \int \left( \dot{\mathbf{n}}_i - \dot{\mathbf{R}}_{ij} \cdot (\dot{\mathbf{n}}_{ij} + \dot{\mathbf{u}}_{ms}) + \dot{\mathbf{P}}_{Cj} \times (\dot{\mathbf{t}}_{ij}) \right) d\tau \\
&= \mathbf{C}_i \mathbf{I}_i \cdot \dot{\omega}_i = \int \left( \dot{\mathbf{n}}_i - \dot{\mathbf{R}}_{ij} \cdot (\dot{\mathbf{n}}_{ij} + \dot{\mathbf{u}}_{ms}) + \dot{\mathbf{P}}_{Cj} \times (\dot{\mathbf{t}}_{ij}) \right) d\tau \\
&= \mathbf{C}_i \mathbf{I}_i \cdot \dot{\omega}_i = \int \left( \dot{\mathbf{n}}_i - \dot{\mathbf{R}}_{ij} \cdot (\dot{\mathbf{n}}_{ij} + \dot{\mathbf{u}}_{ms}) + \dot{\mathbf{P}}_{Cj} \times (\dot{\mathbf{t}}_{ij}) \right) d\tau
\end{align*}
\] (178)

Algebraic elimination of a number of terms from (179) using (170) - (173) and (175) - (177) and integration of (179) yields

\[
\begin{align*}
\dot{\mathbf{r}}_{cj} + \dot{\mathbf{r}}_{ms,j} \times \dot{\mathbf{t}}_{ij} &= \mathbf{C}_i \mathbf{I}_i \cdot \dot{\omega}_i \\
&= \int \left( \dot{\mathbf{r}}_{ms,j} \times \dot{\mathbf{t}}_{ij} \right) d\tau \\
&= \int \left( \dot{\mathbf{r}}_{cj} \times \dot{\mathbf{t}}_{ij} \right) d\tau \\
&= \mathbf{C}_i \mathbf{I}_i \cdot \dot{\omega}_i
\end{align*}
\] (180)

where \( \dot{\mathbf{t}}_{ij} \) is the reaction impulse at joint \( \{i\} \).

To obtain \( \dot{\mathbf{t}}_{ij} = 0 \), the right side of (180) must be zero. In the case where the rotational joints are free (no friction is also assumed), \( D = 0 \). Due to the quasi-static assumption, \( C = 0 \).

For a robot with N links two cases are possible:
i) The joint is at the last link of the robot \( \{ i = N \} \) - then \( B = 0 \). Using (135) and (152) the following applies

\[
C_i I \cdot \omega_i = \dot{r}_{cop,i} \times m_i \cdot (\dot{r}_{i} \times \dot{r}_{C,i})
\]

so, for (180) to apply the following must hold

\[
\dot{r}_{i} \times m_i \cdot (\dot{r}_{i} \times \dot{r}_{C,i}) = 0
\]

or in other words, \( \dot{r}_{i} \) must be zero and the impact occurs at the CoP. If this is not the case, reaction forces develop, as already discussed in Section 5.3.

ii) In the general case where \( i \neq N \), the following must apply

\[
\left[ \left( \dot{r}_{cop,i} + \dot{r}_{i} \right) \times m_i \cdot (\dot{r}_{i} \times \dot{r}_{C,i}) - C_i I \cdot \omega_i \right] - \dot{r}_{dist,i} \times \dot{r}_{i} \cdot i \Omega_{int} = 0
\]

In this case \( A - B = 0 \). This means that in case simultaneously the reaction from joint \( \{ i + 1 \} \) and an impact force \( f_{imp,j} \) act directly on link \( \{ i \} \), then it is possible to eliminate the reaction at joint \( \{ i \} \) if \( A = B \). However a more common and thus interesting situation is the one at which the impact force acts on the end effector (link N) only, and the previous rotational joints \( \{ i < N \} \) must cope with the reaction forces which propagate from their successive joints \( \{ i + 1 \} \). In this case it is easy to see that if the joint \( \{ i + 1 \} \) is on the CoP of link \( \{ i \} \) then \( \dot{r}_{dist,i} = \dot{r}_{i} = 0 \) and thus \( A = B = 0 \).

The above analysis concludes that in order to minimize reaction forces on joint bearings, it is best to position the revolute joints on the CoP of the previous link, while the impact should occur at the CoP of the last link.

Summarizing the previous analysis and setting \( D \neq 0 \), (180) can be zeroed if the following applies

\[
\dot{r}_{i} \times m_i \cdot (\dot{r}_{i} \times \dot{r}_{C,i}) + \int \left[ \dot{r}_{i} \cdot i \Omega_{int} \cdot n_{i} \right] = 0
\]

where \( \dot{r}_{i} \) is the distance of the joint \( \{ i + 1 \} \) (or the IP for the last link, normally this is the point of the end effector) from the CoP of link \( \{ i \} \). In case link \( \{ i \} \) is subject to several impacts including the reaction of joint \( \{ i + 1 \} \), (184) is valid by using the vectorial sum of all impact forces at a distance \( \dot{r}_{i} \) and a corresponding external momentum which will be added in the integral.
5.6.2 Impact Compensation using CoP (IC$^2$)

In case the rotational joints are not free, (184) can be used to examine if the reaction forces can be reduced by motor actuation. First the case where the joints are completely frictionless is examined. The $^i n_i$ is substituted with $^i n_{\text{cop},i}$ for convenience, therefore from (184) the following should apply

$$^i n_{\text{cop},i} = -^i r_i \times m_i \left( ^i \omega_i \times ^i r_{c,j} \right) + \int \left[ ^i R_{\text{rot},i} \cdot ^{i+1} n_{i+1} \right]$$

(185)

Derivation of (185), noting that for this small fraction of time only the angular velocity and the impulse are affected, gives

$$^i n_{\text{cop},i} = -^i r_i \times m_i \left( ^i \omega_i \times ^i r_{c,j} \right) + ^i R_{\text{rot},i} \cdot ^{i+1} n_{i+1}$$

(186)

where it is repeated for convenience that $^i r_i$ is the distance of the joint $\{i+1\}$ or the IP from the CoP of link $\{i\}$. Note that if $i = N$, therefore $\{i\}$ is the last link on the robotic arm, then (186) is simplified

$$^i n_{\text{cop},i} = -^i r_i \times m_i \left( ^i \omega_i \times ^i r_{c,j} \right)$$

(187)

whereas if $i \neq N$ but joint $\{i+1\}$ is located on the CoP of link $\{i\}$, then

$$^i n_{\text{cop},i} = ^i R_{\text{rot},i} \cdot ^{i+1} n_{i+1}$$

(188)

Equations (186) - (188) provide the necessary motor torque in order to compensate an impact which occur on a point different than CoP at the last link and to compensate the reaction forces propagation by the successive joints. Note that in the case of the space robot base an equal torque to the torque applied to the next joint must be applied by the actuators of the base, otherwise due to the dynamic coupling and the reactive motion of the base, reaction forces are developed on joint $\{1\}$ which back-propagate. That is the controller must be applied to all rotational joints including the base. Naturally (186) - (188) cannot provide any compensation for the component of the force whose line of action passes through the joint position.

Additionally it is important to consider than in order for (186) - (188) being effective, and regarding the fact that the impact is a process with fast dynamics, it is necessary to compensate joint friction. Otherwise the compensation torque based on the CoP will not add or subtract the necessary amount of moment, because the friction will create an unwanted moment opposing the motion induced by the motor. Many friction models have been proposed in the literature, such as [5] and [119]. To this end the motor must apply a friction compensation torque in each joint, [114].
More specifically, the motor of each joint must provide a torque \( \tau_i \) which generally is of the form

\[
\tau_i = \tau_{\text{cop},i} + \tau_{\text{fr},i}
\]  

(189)

where \( \tau_{\text{cop},i} \) is the control torque already calculated in (186) - (188) and \( \tau_{\text{fr},i} \) is the compensation for the joint friction.

The proposed controller acts during the impact, whereas after the zeroing of the external impact force (and thus the impact reaction on the joints), the control system switches to the normal system controller (e.g. model based controller which tries to move the end effector to a particular position), Figure 5-8.

In order to implement the IC\(^2\) controller the following scheme is proposed. Let a space robot with equation of motion

\[
\tau = M \cdot \ddot{q} + V(q, \dot{q}) + J^T \cdot F
\]  

(190)

where \( q \) is the vector of joint variables, \( \tau \) is the vector of all actuator forces and torques, \( M \) is the configuration dependent mass matrix, \( V(q, \dot{q}) \) is the vector of nonlinear velocity terms and \( J^T \cdot F \) corresponds to the effect of external impact forces \( F \) to each joint.

The proposed IC\(^2\) controller acts during the impact, whereas after the zeroing of the external impact force (and thus the impact reaction on the joints), the control system switches to the normal system controller (e.g. model based controller which tries to move the end effector to a particular position), Figure 5-8.

Thus the proposed procedure is two-part. In the beginning the robot motion is controlled e.g. by a model based controller, where an acceleration \( \ddot{q}^* \) is calculated

\[
\ddot{q}^* = \ddot{q}_d + K_d \cdot (\ddot{q}_d - \ddot{q}) + K_p \cdot (q_d - q)
\]  

(191)

where \( q_d, \dot{q}_d, \ddot{q}_d \) are the desired position, velocity and acceleration of all degrees of freedom in vectorial form and \( K_d, K_p \) are the gain matrices for derivative and proportional control. Therefore the necessary torques are given by

\[
\tau = \hat{M} \cdot \ddot{q}^* + \hat{V}(q, \dot{q})
\]  

(192)

where \( \hat{M}, \hat{V} \) are matrices with the estimates of mass and nonlinear velocity terms. The exact procedure of model based control is out of the scope of this work, the interested reader may consult
the literature, [25]. As the impact occurs, the torques of the motors are substituted by (189) which in turn are calculated by (186) - (188).

Figure 5-8. Block diagram of Impact Compensation using CoP (IC$^2$).

After each impact, the systems under impact are separated. Then the chaser will reach again the target (due to the inertia of the masses and their initial relative velocity) using (192) and another impact will occur. This can be repeated several times, [30], until the impacts are below a threshold where another control scheme (e.g. Impedance Control) can be used effectively. The proposed IC$^2$ controller can be used for all these impacts in order to reduce their negative results: (i) reduce the impact reaction forces on the joints and (ii) reduce the tendency to separate the chaser from the target.

As the impact is a process with fast dynamics, it is necessary to have a high performance motor drive in order to incorporate fast torque control. It is of high importance the control calculations to be as few and as fast is possible – it is of no meaning to develop a control algorithm which would require considerable time for calculations, comparable to the duration of the impact. The proposed method requires few and fast calculations, thus it is not computationally difficult to get implemented. By design, values as lengths, masses and inertia are known therefore in order for (186) - (188) to get implemented, an encoder on each joint and a force sensor on the last link N (to detect the impact) is required.
A final word relating to the knowledge of the IP point is considered necessary. For (186) and (187), \( r_i \) (the distance of the joint \( \{i+1\} \) or the IP from the CoP of link \( \{i\} \)) is required. In practice one can establish two cases: (a) the IP is known a priori and (b) the IP is unknown. In the first case the IP is known by design (or by a priori planning); as a result the equations can be used with this known value and any discrepancies due to inaccurate IP calculation can be low – the accepted inaccuracies can be estimated by the procedure of Sec. 5.5. In the second case a system which will detect quickly the IP to feed its value to the equations is required. Many researchers are working on the field and some notable works can be found in the literature, [7], [18] and [19]. Naturally these methods, even though they are fast and can be used in real time, add a small delay to the calculations, however this is out of the scope of this work.

5.7 Implementation Guidelines

As in space robotic applications revolute joints are used commonly, here the focus is on such joints. Hence a number of guidelines to exploit the CoP using revolute joints are presented. First the case of free joints or joints which can be disengaged from their actuation rendering them essentially free, are examined. The following guidelines apply:

i. An impact should occur as near as possible to the measured CoP and at normal angle with respect to longitudinal axis which is defined by the RA (2-D Case) or RP (3-D Case) to the IP. To this end, the robotic system must prepare itself for the impact. The equations which describe the use of CoP are summarized by (141), (155), (156) and (160).

ii. Axes of revolute joints should be normal to each other at the moment of the impact, see (129) and connected at their CoP, see (180) (assuming the latter property is achievable).

iii. To filter an impact in 2-D systems, two revolute joints are needed, while in the 3-D case, three revolute joints are needed, each corresponding to each component of the impact force.

iv. In the presence of uncertainties the configuration with successive normal links and normal impact angle gives the best results, due to the fact that this configuration can gradually filter most of the residue components of the impact force.

Next the case of joints engaged with actuation is examined. For such joints the following guidelines also should apply:

v. In case of an impact at a point different from the CoP, an additional torque, computed using (186) - (188), should be applied to all rotational joints including the base.
vi. In conjunction with guideline (v), the controller should apply the necessary torque to cancel friction.

5.8 Simulation Results

5.8.1 Planar Space Robot with Free Joints

The first set of simulations refers to planar systems “A” and “B”, Figure 5-9, which consists of a thruster-equipped base, able to make $x - y$ translational planar and rotational motions. Table 5-1 and Table 5-2 displays the physical properties of the systems, including the position of CoP for the two rotational links. Simulations were run by changing various parameters: (a) impact position on link 5, (b) impact angle, (c) position of joint 5 on link 4 and (d) initial angles of joints $q_2$ and $q_3$. Both joints are free to rotate. The impact duration is 0.01s, and the magnitude is set equal to 10kN. In order to make more convenient the comparison of impact configurations, forces were plotted instead of impulses. Also, the plots present the force components on the local coordinate frame of each joint, i.e. the normal component of the impact force is parallel to the $y_i$ axis of the local CS of link $\{i\}$.

Table 5-1. Physical characteristics of planar system “A”.

<table>
<thead>
<tr>
<th>Link</th>
<th>Mass (kg)</th>
<th>Inertia(kg-m$^2$)</th>
<th>Joint length (m)</th>
<th>CoM (m)</th>
<th>CoP (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>66.67</td>
<td>1</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>3.33</td>
<td>1</td>
<td>0.5</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>2.5</td>
<td>1</td>
<td>0.5</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 5-2. Physical characteristics of planar system “B”.

<table>
<thead>
<tr>
<th>Link</th>
<th>Mass (kg)</th>
<th>Inertia(kg-m$^2$)</th>
<th>Joint length (m)</th>
<th>COM (m)</th>
<th>CoP (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>50</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>3.33</td>
<td>1</td>
<td>0.3</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>2.5</td>
<td>1</td>
<td>0.3</td>
<td>0.72</td>
</tr>
</tbody>
</table>
Figure 5-9. The 2-D Space System and its DOFs used in simulations (a) Type “A” and (b) “Type “B”.

Figure 5-10 presents the reaction forces after the impact at link 5 of system Type “A”: (a) on joint 5 changing the point of impact on link 5, (b) on joint 5 changing the angle of impact on CoP of link 5, and (c) on joint 4 when the impact angle is 60° and the position of the rotational joint 5 varies on the link 4 (which is always normal to link 5). As shown in Figure 5-10a, the local reaction force is almost eliminated when the impact is located on the CoP, whereas in Figure 5-10b, the reaction force is almost eliminated when the impact angle is normal to link 5. Note that after the impact (i.e. after the 1.01s) the reaction forces may vary, however this is solely due to the post-impact free motion and the consequent dynamic coupling between the bodies. Similarly in Figure 5-10c, the reaction force in joint 4 is almost zeroed when the location of the joint 5 is in the CoP of link 4. These results validate the use of CoP notion in order to minimize the reaction impulses. More specifically it can be seen that when the impact occurs on the CoP with a normal angle, the reactions during the impact are minimized, thus the system has a more smooth behaviour. Similarly when the impact is not normal, the configuration where the joint 5 is on the CoP of link 4 filters the component of the impact which cannot be minimized from joint 5, that is
the component which is parallel to link 5. On the other hand when the impact does not occur on the CoP and/or with normal angle, there is always a residue reaction force propagating to the next link and eventually the robot base.

**Figure 5-10.** Reaction forces on local coordinates of joint 5 of planar system Type “A” for alternating a) different impact point and b) different impact angle and c) reaction forces on local coordinates of joint 4 for alternating joint 5 position.
Similar results can be seen for planar system Type “B” in Figure 5-11.

**Figure 5-11.** Reaction forces on local coordinates of joint 2 of planar system Type “B” for alternating a) different impact point and b) different impact angle and c) reaction forces on local coordinates of joint 1 for alternating impact angle.
Figure 5-12 presents the motion of the robotic system with free joints after impact for (a) a non-ideal configuration and (b) an ideal one with the two links normal to each other, and the force acting at the CoP normal to the final link. Both simulations last for 1.2s and refer to the planar system Type “A”. At final time, the CoM of base has translated 0.22m in the first case and 0.12m in the second case. The relatively fast link motion is due to the applied impact force and the system mass properties, which were selected for illustrative purposes. The response will be slower or faster depending on these values, but qualitatively similar. Again the use of the ideal configuration allows the system to filter the reactions, thus the magnitude of the force which propagates to the robot base is less which in turn translates the system for a smaller distance from the starting point in contrast with the distance travelled when the impact occurs on the non-ideal configuration.

Figure 5-12. Motion snapshots of the 2-D system with free joints following impact. (a) non-ideal impact configuration, (b) ideal impact configuration.
5.8.2 3-D 3R system

The next set of simulations presents a 3R PUMA-like robot with free joints. The joint axis of the first joint is at 90° with respect to the second one, which is parallel to that of the third joint, see Figure 5-13. That is, it inherently satisfies guideline (a). The properties of the system are presented in Table 5-3. The manipulator is fixed on a large base, e.g. ISS. The reaction forces on the bearings of joints and the reaction on the base are of interest. The impact force is 
\[ F_{imp} = [1 1 1]^T kN, \| F_{imp} \| = 1.732 kN, \] with duration 0.01s.

Figure 5-14 presents the results for two different setups; the blue dotted line presents a configuration, where all joints are located at the CoP of links (denoted by C, C, C) and the initial angles are \((q_1,q_2,q_3) = (0^\circ,0^\circ,90^\circ)\). The red solid line presents a random setup with joints at the tip of each link (denoted by 1, 1, 1) and initial angles \((q_1,q_2,q_3) = (0^\circ,30^\circ,45^\circ)\).

![Figure 5-13. The 3-D RRR robotic system under evaluation.](image)

Table 5-3. Properties of 3R-3-D system under evaluation.

<table>
<thead>
<tr>
<th>Link</th>
<th>Mass (kg)</th>
<th>Inertia(kg-m²)</th>
<th>Joint length (m)</th>
<th>CoM(m)</th>
<th>CoP (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>2.5</td>
<td>1</td>
<td>0.3</td>
<td>0.417</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.5</td>
<td>1</td>
<td>0.15</td>
<td>0.667</td>
</tr>
</tbody>
</table>

Table 5-4 presents the magnitude of reaction forces per joint for some characteristic configurations and impact points or joint positions. As both Figure 5-14 and Table 5-4 show, the use of the CoP has advantages. In Figure 5-14 one may observe that at joint 1 (base) the forces developed are much lower when the guidelines are satisfied. However they are not zero due to the dynamic...
coupling between links and joints 1 and 2. This can be seen also in the third and fourth cases of Table 5-4, where the third link has the same setup (impact on the CoP) but different magnitudes have been found. Still the configuration with normal links and impacts on the CoP minimizes the impulse reactions at the base.

Figure 5-14. Reaction forces for the 3R in two distinct configuration cases.

Table 5-4. Max reaction forces (absolute values) as a function of configuration & joint location.

<table>
<thead>
<tr>
<th>Config.</th>
<th>1st Case</th>
<th>2nd Case</th>
<th>3rd Case</th>
<th>4th Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint Ang.</td>
<td>(0°, 0°, 0°)</td>
<td>(0°, 0°, 0°)</td>
<td>(0°, 0°, 90°)</td>
<td>(0°, 0°, 90°)</td>
</tr>
<tr>
<td>Joint Loc.</td>
<td>(1, 1, 1)</td>
<td>(1, C, C)</td>
<td>(1, 1, C)</td>
<td>(C, C, C)</td>
</tr>
<tr>
<td>Joint 1 (N)</td>
<td>1583</td>
<td>1390</td>
<td>419</td>
<td>310</td>
</tr>
<tr>
<td>Joint 2 (N)</td>
<td>1520</td>
<td>1337</td>
<td>355</td>
<td>220</td>
</tr>
<tr>
<td>Joint 3 (N)</td>
<td>1307</td>
<td>1260</td>
<td>838</td>
<td>997</td>
</tr>
</tbody>
</table>

5.8.3 Planar Space Robot with IC²

In order to examine the performance of the IC² controller, a set of simulations was run based on the planar robot of Figure 5-9 (both types). However, in this case the impact occurred on the end-effector at time t=1s with duration 0.1s and magnitude in all cases 100N. In the first case of Type “A”, three different configurations were examined: (i) A configuration with links, joints and impact positions at random positions and free joints (in this case \( q_2 = -80° \), \( q_3 = 85° \), position of
joint 5 at 0.9m and IP 0.85m) (ii) A configuration with the same setup as in case (i) but this time with the use of the IC\textsuperscript{2} and finally, (iii) A configuration with all links normal and joint and impact positions at the end of each link and the use of the IC\textsuperscript{2} (that is \( q_2 = -90^\circ, q_3 = 90^\circ \), position of joint 5 at 1m and IP 1m).

For the application of IC\textsuperscript{2} (186)-(188) were used and the friction compensation was supposed to be ideal in order to examine the validity of the impact compensation part of the controller. As it can be seen in Figure 5-15, the use of IC\textsuperscript{2} reduces the reaction forces, without requiring large computational burden – this is critical as it can be applied within the duration of the impact. However it is important that the impact will occur with an ideal configuration (links normal to each other, impact angle normal, impact point near the CoP), as it will reduce motor torques requirements considerably – large deviations will increase the force components that cannot be compensated (i.e. components parallel to the longitudinal axis of the link) or the motor torque requirements to saturation levels.

In this case of Type “B” planar robot, the impact occurred on the end-effector at time \( t=0.5s \) with duration 0.1s and magnitude in all cases 100N. For the application of IC\textsuperscript{2} (186)-(188) were used and the friction compensation was supposed to be ideal in order to examine the validity of the impact compensation part of the controller.

During the simulations, the robot has the two links in normal configuration (thus \( q_1 = -90^\circ \) and \( q_2 = 90^\circ \)). Joint 1 is located at the tip of link 1 (thus further from the CoP) and the impact occurs at various points on link 1, with normal direction to the longitudinal axis of link 2, as the guidelines suggest. As it can be seen in Figure 5-16 where the impact occurs at the tip of link 2 (thus the IP is at 1m), the use of IC\textsuperscript{2} reduces the reaction forces, without requiring large computational burden – this is critical as it can be applied within the duration of the impact. However it is important that the impact will occur with an ideal configuration (links normal to each other, impact angle normal, impact point near the CoP), as it will reduce motor torques requirements considerably – large deviations will increase the force components that cannot be compensated (i.e. components parallel to the longitudinal axis of the link) or the motor torque requirements to saturation levels.

Finally in Figure 5-17 shows the required motor torques during impact at different IPs. As the IP is further from the CoP with respect to the CoM the required torque has clockwise direction, while as it the IP is between the CoP and the CoM it becomes counter-clockwise (note that the direction of the torque depends on the direction of the impact force). It is interesting that near the CoP the torque is almost zero (not exactly zero, as the links start to rotate and thus a torque is applied).
Figure 5-15. Demonstration of the IC² controller for planar system Type “A”: (a) Reaction forces on joint 4, (b) Reaction forces on joint 5 and (c) Motor torques on all joints for case (iii).
Figure 5-16. Demonstration of the IC² controller for planar system Type “B”: (a) Reaction forces on joints 1 & 2 if the robot has free joints or uses IC², (b) Motor torques on all joints and base’s reaction wheel for the case the IC² is used.
<table>
<thead>
<tr>
<th>Joint 2</th>
<th>Joint 1</th>
<th>Base</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Motor Torque (Nm)</strong></td>
<td><strong>Motor Torque (Nm)</strong></td>
<td><strong>Motor Torque (Nm)</strong></td>
</tr>
<tr>
<td>IP 0.5m</td>
<td>IP 0.5m</td>
<td>IP 0.5m</td>
</tr>
<tr>
<td>IP CoP</td>
<td>IP CoP</td>
<td>IP CoP</td>
</tr>
<tr>
<td>IP 1m</td>
<td>IP 1m</td>
<td>IP 1m</td>
</tr>
</tbody>
</table>

**Figure 5-17.** Motor torques on all joints and base’s reaction wheel using IC² for different IP for planar system Type “B.”
6 Conclusions and Future Work

6.1 Conclusions

The research in this work focuses on mitigating the effects of impacts between an orbital robotic system (Chaser) and a (usually uncooperative) satellite or space debris (Target). To this end, in this work three particular aspects of the impact process are examined: effective simulation of impacts, correct modelling of impacts of free floating systems and analysis of the effects of inertia and stiffness parameters and mitigation of impact effects by exploiting the Center of Percussion.

The research studied first the correct methods of modeling impacts. The current methods, such as the rigid body theory, the viscoelastic theory and the FEM, although used in many applications, have their limitations. A fast and accurate method that could describe the deformations that may occur during an impact was necessary. By using concepts from other fields of engineering, a novel impact model was proposed and developed which displays viscoplastic characteristics. This viscoplastic model shows qualitative similarity with experimental results found in the literature. This model can efficiently describe a large number of interactions that occur in robotics, not only in space but also in terrestrial applications. At the same time a parameter named Coefficient of Permanent Deformation has been introduced, which describes the deformations that can occur on a viscoplastic material, taking into account complex behaviours like compaction and cratering.

The current methods of describing the impacts between systems of bodies in free-floating environment are analyzed. It is shown that current methods are simple due to the fact that usually one of the bodies is much larger than the others. It can be found analytically that during impacts what is important is not the absolute value of the masses of the systems that come into contact, but the ratio of the masses involved. By using simple but realistic assumptions, an analysis is developed which shows the dependence on mass ratios. For this reason the ratio of effective masses which can quantitatively describe the behaviour of systems under impact is introduced, taking into account the ratio of all masses during any impact. By using this ratio, a way to assess the post-impact relative velocity knowing only the pre-impact relative velocity is presented. Using these elements, a method to determine the minimum relative velocity before impact in order to achieve docking was proposed.

Additionally, ways to mitigate the effects of impacts during operations like docking were examined. For this reason, a known notion from dynamics, the Center of Percussion (CoP) is used. After a brief introduction of the CoP for 2-D, the analytical extension of the CoP concept in 3-D is presented. It is shown that one can analyse the CoP in 3-D similarly as in 2-D (plane), if and only
if some specific characteristics of the body under impact apply. A non-dimensional sensitivity analysis of the CoP effects follows. More specifically, the effects of inaccurate parameters in the systems under impact using the CoP theory are presented and methods are proposed for their mitigation during the design of a manipulator and/or its controller. Finally the Impact Compensation using the CoP (IC²) controller, which takes advantage of the CoP theory to reduce (theoretically eliminate) the reaction forces on the manipulator joints is proposed. A number of simulations demonstrate the validity of the concepts.

6.2 Future Work

This work presented a study of the impacts in free floating systems for tasks like grasping or docking. Besides the presented results, a number of open issues in this field of robotics exists.

The proposed viscoplastic model has been tested in simulations of various applications such as in space or in the demanding task of impacts during foot-terrain interactions. This model can be further exploited by adding the effects of friction and by assessing its performance to even more complex environments like those of granular soils. This would require both analytical developments and also correlation with experimental results. Particularly interesting is to determine experimentally the values of the Coefficient of Permanent Deformation for various materials and not rely on existing experimental results. Additionally, a method to determine the deformation of each body during an impact (and not the total deformation of both bodies under impact only) would add further to the realism of the model and understanding on the physical phenomena that undergo during impacts. An analytical determination of the energetic coefficient of restitution for the impact model would be also useful. Incorporation of all these results to a method for connecting the values of soil characteristics used in static experiments (like cohesion) with values of soil characteristics in dynamic experiments (i.e. stiffness and damping) would provide a great research tool for simulating efficiently various soils under different impact types.

The modeling of impacts between multibody systems gave rise to several interesting insights for the behaviour of the robotics systems in space as they come into contact. However a generalisation of the analysis by incorporating the effects of springs and dampers would further deepen the understanding of impacts. Another interesting point would be a generalisation of the coefficient of effective masses for use on multibody systems with more than two bodies per system, when there are several points with compliance of equal magnitude. The development of analytical methods to derive the minimum and maximum velocities of impact in order to achieve latch, and the non-dimensionalization of the parameters which are included in the process is also a work that has further potential. Whether this method will be analytical or numerical however is an
open question. The development of the CSL space emulator will allow to study the impact behaviour and test the theoretical analysis, fine-tuning it.

Finally there was an extensive analysis of the CoP in 2-D and 3-D, the study of their parametric sensitivity, and the development of IC$^2$ controller. This field seems to have great potential not only in space but also in terrestrial applications. It would be interesting to find loci with same non-dimensional impulse percentages (apparently non-zero percentages) in the 3-D case. Next the controller could be further studied focusing on various effects that have not been examined here, such as friction, gears and non-backdrivability (or limited backdrivability) of many manipulators, control digitization, etc.
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Appendix A

A-1 Effective Values, Spring and Damping Constants

During the development of various engineering equations, it is common to encounter equations of the following type

\[ <\text{value}_1>_{\text{eff}} = \frac{<\text{value}_1> \cdot <\text{value}_2>}{<\text{value}_1> + <\text{value}_2>} \quad (193) \]

where \(<\text{value}_i>, i=1,2\) is a value of some kind.

Especially during the analysis of impacts one can find (193) during the description of the coupling or interaction of masses. For this reason this equation shall be analysed further. For clarity the \(<\text{value}_i>\) shall be replaced by mass. Therefore for two masses \(m_1\) and \(m_2\) which are under impact or connected by a compliant element, their effective mass is

\[ m_{\text{eff}} = \frac{m_1 \cdot m_2}{m_1 + m_2} \quad (194) \]

It is more interesting to see the behaviour of \(m_{\text{eff}}\) if in (194) \(m_1\) is replaced by

\[ m_1 = \lambda \cdot m_2 \quad (195) \]

where \(\lambda\) is a ratio of the two masses. Using (195) in (194) it can be found

\[ m_{\text{eff}} = \frac{\lambda \cdot m_2}{\lambda + 1} \quad (196) \]

As it can be seen in (196), as the ratio \(\lambda\) tends to zero the effective mass tends to zero, and as it tends to infinity the effective mass tends to \(m_2\), thus

\[ m_1 << m_2 \Rightarrow \lim_{\lambda \to 0} m_{\text{eff}} = 0 \]
\[ m_1 >> m_2 \Rightarrow \lim_{\lambda \to +\infty} m_{\text{eff}} = m_2 \quad (197) \]

The derivative of (196) is

\[ \frac{dm_{\text{eff}}}{d\lambda} = \frac{m_2}{(\lambda + 1)^2} > 0 \quad (198) \]

thus (196) is strictly increasing. Finally interestingly for \(\lambda = 1\)

\[ \lambda = 1 \Leftrightarrow m_1 = m_2 = m \Leftrightarrow m_{\text{eff}} = \frac{m}{2} \quad (199) \]
Due to (197) and (198), it can be concluded that the maximum value of the effective mass cannot be higher than the smaller mass (the conclusion applies also to any other effective value). This last remark plays an important role in this work.

**A-2 Springs and Dampers in series and in parallel**

If two springs with spring constants $k_1, k_2$ are connected in series the total spring constant $k_{eq}$ is

$$k_{eq} = \frac{k_1 \cdot k_2}{k_1 + k_2} \quad (200)$$

which means that in case $k_1 \gg k_2$ then $k_{eq} = k_2$, or in other words the softer spring dominates.

If two springs with spring constants $k_1, k_2$ are connected in parallel the total spring constant $k_{eq}$ is

$$k_{eq} = k_1 + k_2 \quad (201)$$

which means that in case $k_1 \gg k_2$ then $k_{eq} = k_1$, or in other words the harder spring dominates.

If two dampers with damping coefficients $c_1, c_2$ are connected in series the total spring constant $c_{eq}$ is

$$c_{eq} = \frac{c_1 \cdot c_2}{c_1 + c_2} \quad (202)$$

which means that in case $c_1 \gg c_2$ then $c_{eq} = c_2$, or in other words the damper with the lower coefficient dominates.

If two dampers with damping coefficients $c_1, c_2$ are connected in parallel the total spring constant $c_{eq}$ is

$$c_{eq} = c_1 + c_2 \quad (203)$$

which means that in case $c_1 \gg c_2$ then $c_{eq} = c_1$, or in other words the damper with the higher coefficient dominates.
Appendix B

In this appendix the Simulink block diagrams used in Sec. 4 and 5 are briefly presented. In Figure B-1 the two-bodies systems of Sec. 4.5 are depicted by separate blocks, Figure B-2. Their interaction forces are calculated, depending on their positions and velocities according to Figure B-3.

In Figure B-4 the simulink block diagram for the systems in Sec. 5.8 is presented.

Figure B-1, Simulink block diagram of the simulated interaction of the two two-bodies systems.
Figure B.2. Simulink block diagram of the Chaser two-body system. A similar block diagram for the Target system has been developed.
Figure B-3. Simulink block diagram for the calculation of the interpenetration forces during impact.
Figure B-4. Simulink block diagram used in the CoP analysis. By changing the MATLAB Function the different systems have been described.
Appendix C

C-1 Dynamic Model of the Planar Space System

In this section the matrices for the dynamic model of the planar system are presented.

% Mass Matrix

\[
M = \begin{bmatrix}
m1 + m2 + m3, \\
0, \\
-1 * m2 * \sin(x(3) + x(4)) * \text{link2} - (m2 + m3) * \sin(x(3)) * \text{link1}, \\
-1 * m3 * \sin(x(3) + x(4) + x(5)) * \text{link2}, \\
-1 * m3 * \sin(x(3) + x(4) + x(5)) * \text{link2}, \\
om1 + m2 + m3, \\
0, \\
m2 * \cos(x(3) + x(4)) * \text{link2} + (m2 + m3) * \cos(x(3)) * \text{link1}, \\
m3 * \cos(x(3) + x(4)) * \text{link2} + (m2 + m3) * \cos(x(3)) * \text{link1}, \\
-1 * m2 * \sin(x(3) + x(4)) * \text{link2}, \\
m3 * \cos(x(3) + x(4)) * \text{link2} + (m2 + m3) * \cos(x(3)) * \text{link1}, \\
\end{bmatrix}
\]

% Transposed Jacobian Matrix

\[
JT = \begin{bmatrix}
-\cos(x(3) + x(4) + x(5)), \sin(x(3) + x(4) + x(5)), 0, \\
0, \\
-\cos(x(3) + x(4)), \sin(x(3) + x(4)), 0, \\
\end{bmatrix}
\]
\[\begin{align*}
-\cos(x(3)), \sin(x(3)), 0; \\
-\sin(x(3)) + x(4) + x(5), -\cos(x(3) + x(4) + x(5)), 0, ... \\
-\sin(x(3) + x(4)), -\cos(x(3) + x(4)), 0, -\sin(x(3)), -\cos(x(3)), 0; \\
-\sin(x(5)) \times \text{link2} - \sin(x(4) + x(5)) \times \text{link1}, ... \\
-rflk5 - \cos(x(5)) \times \text{link2} - \cos(x(4) + x(5)) \times \text{link1}, ... \\
-1, -\sin(x(4)) \times \text{link1}, ... \\
-rflk4 - \cos(x(4)) \times \text{link1}, -1, ... \\
rflk3y, -rflk3x, -1; \\
-\sin(x(5)) \times \text{link2}, -rflk5 - \cos(x(5)) \times \text{link2}, -1, ... \\
0, -rflk4, -1, 0, 0, 0; \\
0, -rflk5, -1, 0, 0, 0, 0, 0; \\
\end{align*}\]

% V Matrix

\[ V = [ 0, 0, -1 \times ((m2 + m3) \times \cos(x(3)) \times \text{link1} + m3 \times \cos(x(3) + x(4)) \times \text{link2}) \times x(8) - 2 \times m3 \times \cos(x(3) + x(4)) \times \text{link2} \times x(9) - m2 \times \cos(x(3) + x(4)) \times \text{link1} \times x(8) + ... \\
2 \times x(9) - m3 \times \cos(x(3) + x(4) + x(5)) \times \text{link2} \times x(8) + 2 \times (x(9) + x(10))), ... \\
(-1) \times m2 \times \cos(x(3) + x(4)) \times \text{link2} \times x(9) - m3 \times (\cos(x(3) + x(4)) \times \text{link2}) \times x(9) + \cos(x(3) + x(4) + x(5)) \times \text{link2} \times x(9) + ... \\
0, 0, ... \\
-1 \times (m3 \times \sin(x(3) + x(4)) \times \text{link2} + (m2 + m3) \times \sin(x(3)) \times \text{link1}) \times x(8) - 2 \times m3 \times \sin(x(3) + x(4)) \times \text{link2} \times x(9) - m2 \times \sin(x(3) + x(4)) \times \text{link1} \times x(8) + 2 \times (x(9) + x(10))), ... \\
(-1) \times m2 \times \sin(x(3) + x(4)) \times \text{link2} \times x(9) - m3 \times (\sin(x(3) + x(4)) \times \text{link2}) \times x(9) + \sin(x(3) + x(4) + x(5)) \times \text{link2} \times x(9) + ... \\
0, 0, ... \\
-2 \times \sin(x(3)) \times \text{link2} \times x(8) - m3 \times \sin(x(4) + x(5)) \times \text{link2} \times x(9) + m3 \times \sin(x(4) + x(5)) \times \text{link2} \times x(10) ; \\
0, 0, ... \\
-1 \times \text{link1} \times (m3 \times \sin(x(4) + x(5)) \times \text{link2}) \times x(9) + m3 \times \sin(x(4) + x(5)) \times \text{link1} \times x(8) ; \\
0, 0, ... \\
-1 \times \sin(x(4) + x(5)) \times \text{link2} \times x(9) + m3 \times \sin(x(4) + x(5)) \times \text{link2} \times x(10) ; \\
0, 0, 0, ... \\
-2 \times \sin(x(4)) \times \text{link2} \times x(9) + m3 \times \sin(x(4)) \times \text{link2} \times x(10) ; \\
-1 \times \sin(x(5)) \times \text{link2} \times x(9) + m3 \times \sin(x(5)) \times \text{link2} \times x(10) ; \\
0, 0, 0, ... \\
m3 \times \sin(x(5)) \times \text{link2} \times x(9), 0] ; \\
\]

\[ C = \begin{bmatrix} 
\end{bmatrix} \]

C-2 Dynamic Model of the 3R System located on ISS

In this section the matrices for the dynamic model of the 3R system are presented.

\[ M = \begin{bmatrix} 
L133 + (m1 \times (\text{link1})^2) + (m2 \times (\cos(q2))^2 \times (\text{link2})^2) + ... \\
(2 \times m2 \times (\cos(q2)) \times \text{link2}) + (m2 + m3) \times (\text{link1}^2) + (2 \times m3 \times (\cos(q2)) \times \text{link1} \times \text{link2}) + ... \\
(m3 \times (\cos(q2))^2 \times (\text{link2})^2) + (2 \times m3 \times (\cos(q2) \times \text{link1} \times (\text{link1} + (\cos(q2) \times \text{link2}))) + ... 
\end{bmatrix} \]
0,...
0;
0,...
$L_{333} + L_{333} + (m_2*(l_{CoM}^2)) + (m_3*(l_{CoM}^3) + \text{other terms}),...$

% Transposed Jacobian Matrix

\[ JT = \begin{bmatrix}
0,...
0,...
-(rfimpx*(cos(q_2+q_3)))-link1-\{cos(q_2)*link2};...
-(sin(q_3))*link2,...
-rfimx-\{cos(q_3)*link2},...
0;...
0,...
-rfimx,...
0; \\
\end{bmatrix} \]

% V Matrix

\[ V = \begin{bmatrix}
-(m_2*(sin(2*q_2))*(l_{CoM}^2)*qdot2)-\{2^*m_2*(sin(q_2))*l_{CoM}+qdot2)-... \\
(2^*m_3*(cos(q_2+q_3))*l_{CoM}+l_{link1}+\{cos(q_2)*l_{link2})*... \\
((sin(q_2))*l_{link2}*qdot2+sin(q_2+q_3)) *l_{CoM}*(qdot2+qdot3)),... \\
0;...
0;...
\end{bmatrix} \]
Appendix D

Pseudocode of the algorithm for the proposed viscoplastic model of Sec. 3.

\textbf{<Set initial conditions>}

\textbf{For} \( k, b \) \textbf{and} \( \lambda^i \) \textbf{from} (60)

\textbf{<For impact instant \( i \) run>}

\textbf{(59) for} \( F_c^i \) \textbf{until} \( \dot{y} = 0 \).

\textbf{<If} \( \dot{y} = 0 \) \textbf{>}

Keep \( y_r^i \) \textbf{and calculate} \( y_f^i \) \textbf{from} (63).

Set \( \lambda^i_r \) \textbf{from} (60)

\textbf{<For impact instant \( i \) run>}

\textbf{(59) for} \( F_r^i \)

\textbf{<If} \( F_r^i = 0 \) \textbf{>}

Impact ends

\textbf{<Elseif} \( F_r^i \neq 0 \) \textbf{and} \( \dot{y} = 0 \) \textbf{>}

Recompression occurs.

\textbf{<Set} \( \lambda_r^{i+1} = \lambda_r^i \)

\textbf{<start over>}

\textbf{<End>
Appendix E

During the upgrade of the CSL space emulator a number of Diploma and Master Theses have been written, while the author was responsible in co-advising and managing the development team. In the next few pages, a brief presentation of the development process will take place. This work was part of the Heracleitus II funding. More information can be found in [3], [99], [100], [123], [124] and [129].

E-1 Concept and Requirements

Addressing the need of developing the new Space Robot according to the standards set by the existing one, [99], led to a conceptual design of a cylindrical, fully autonomous structure floating on the granite table and driven by thrusters. However, the expertise gained through the development process of the previous robot in conjunction with problems revealed during its application, determined additional design parameters and requirements that should be taken into account.

While the shape was predefined, restrictions for similarity on general dimensions were also applied. A major requirement for the new design was to increase the available volume in order to avoid overlaps between various components. Smaller and more efficient parts would have to be employed so as to obtain the same or even better performance, within less space. Correspondingly, a demand was set to maintain regional separation among pneumatics, electronics, batteries and all other subsystems. Obviously, easy assembling and modularity were required, in order to effortlessly replace every individual component, if needed.

Another requirement that came up by the use of the first robot was the trunking task. Wires and CO₂ hoses had to be guided so as to prevent their tangling and to provide protection and immobility. These are qualities that have proven indispensable by experience gained from the previous robot and taken into consideration for the new design. A last requirement was the capability of a replaceable front section in order to accommodate different payloads like manipulators, docking subsystem, or any other experimental device is of particular interest.

As far as the electrical/ electronic subsystem is concerned, and in comparison to the first robot’s design, the requirements included efforts for cost and volume reduction with respect to the first robot, while retaining or extending (if possible) the functionality. Naturally the new hardware should have been compatible with the attach points of the modified chassis. However, no limitations were imposed on the design principles and methodology, neither on the development process.
Electrical/ electronic and software design were performed from the beginning, irrespective of methods employed on the first robot, and only based on the basic requirements which generally define the emulator, namely: (a) power and computation autonomy of the robot, (b) remote control, (c) localization employing an onboard system of optical sensors, (d) localization through wireless communication with an external camera system, (e) thruster and motor control with the respective drives and custom PCBs.

E-2 Developed Robot

E-2.1 Mechanical Design of Base

The mechanical design of the new robot started aiming at creating a structure capable to be built inside institution facilities, and mainly at CSL’s workshop. Structural parts designed for ease of machining and assembly, maintaining a bottom to top procedure which will result to a targeted and robust configuration. The experience gained and the new requirements defined the procedure to be trailed for design and construction.

To begin with, the robot chassis was built out of standard materials (aluminum alloys and Plexiglas), off-the-shelf structural members and general hardware. Particularly, robot’s frame consists of two cylindrical ring type plates which are kept apart using three U channel columns, thus creating a two-floor structure. This design provides approximately 30 lt of available volume to be exploited for subsystems installation, while in its flat top are situated the LEDs for the overhead camera. In more detail, the previously used bottom T base with the air-bearings attached on each vertex of it, was reshaped to a CNC-milled cylindrical plate profile providing increased useful area. Furthermore, the Reaction Wheel (RW) and CO₂ tank pocket were displaced radially and anti-symmetrically, while the latter also received a lowered floor datum under the base bottom surface, allowing for a higher tank and for separating the construction of the several modules, see Figure E-1.

Figure E-1. CAD representation of the bottom base.
Proceeding with the strategy of creating an emulator for increased ease of use, a second design optimization took place. The new principle is the employment of a layout where the various subsystems are situated in distinct modules. The new robot is divided in regions where clusters of servoamplifiers, electronic boards, batteries, pneumatic components and computational/communicational segments are lying together. Every cluster is bordered with Plexiglas planes, which have been designed as shelf, drawer or box and fastened to robot chassis. Special care was paid on achieving quick and simple mounting/removing. Every component part can be reached by pulling out a drawer or by opening a door. As a result the process of maintenance is significantly improved and accelerated due to the excellent accessibility.

Yet, to comply with the requirements of extending emulator capabilities and to accomplish more complex scenarios, the new mechanical design imposed innovations that increased the precision and interchangeability in the payload. Regarding the precision, a new design allowed for the capability to regulate the height of the optical sensors. Particularly, the three optical sensors which are placed at the bottom of robot and provide its position, are now capable to be set in a delimited distance above the granite table, so as to minimize sensor error.

Also, enhancements have taken place in the field of gripping and transporting the emulator. Hooks and custom-made handles designed and constructed to provide the ability of placing/removing the robot on the granite table with smooth movements and without vibrations, hence shaking of sensors has been eliminated. Whereas discussing the interchangeability of the payload, the new front panel base has to be mentioned. This sliding-type base can be fastened if needed at the front plane of the bottom base via screws and it can be used to attach any type of payload. Future experiments employing manipulators, docking system or special sensors can be fulfilled by modulating the base in a manner to carry the appropriate equipment.

In addition to structural upgrades, improvements were made in the area of robot pneumatics. The evolution in the relevant components has been exploited to reduce their volume and increase their effectiveness. In more detail, the formerly used industrial-type high pressure regulator at the top of the tank was replaced with a tiny paintball part called “Palmer Stabilizer” capable to regulate and stabilize gas output. In combination with a coiled remote hose, a quick release coupling and a targeted design concerning siting, resulted in a CO₂ supply system that can be assembled in short time and with no need for any wrenches. Even the CO₂ tank itself was improved, as new gas bottles with fill indicators were purchased and employed. Moreover, major changes were introduced in the thrusting system. Brand new thrusting nozzles fabricated by FESTO as standard components, replaced the earlier custom ones. The nozzle pipe diameter was enlarged and consequently thrusting force was raised. Thruster switching valves were substituted by state of the art models providing response time less than 2ms. Maximum switching frequency
can now reach 330Hz from 130Hz which was previously; accordingly thrusting PWM frequency can also rise.

Finally, the issue of CO$_2$ tube trunking was addressed. Previous experience has shown that cables and pipes can cause troubles during use and maintenance of the robot, since they are intermeshing, pulled and ultimately they are displaced with respect to their correct position/connection. Thus, the new design foresaw this task and special trunking paths were constructed, so as pipes and wires to remain organized and unshakeable. The new robot can be seen in Figure E-2.

![Figure E-2. (a) CAD representation of new robot structure and (b) the current configuration.](image)

### E-2.2 Mechanical Design of Manipulator

The robotic manipulators that were designed for the second robot have two degrees of freedom with two rotary joints each that can rotate about axes perpendicular to the horizontal plane. They are mounted on a chassis that is detachable from the main body of the robot and are capable of folding fully. The ends of the manipulators are properly designed for mounting various tools and/or force and torque sensors. Each manipulator is powered by two DC electric motors with planetary gearboxes. It was deemed best that both of them have to be placed on the chassis of the manipulators, in order to achieve reduced moment of inertia and a favorable center of mass location. Therefore, a transmission system is required for each manipulator’s second joint, which is implemented via timing belts and pulleys. The assembly of the two manipulators and the chassis comprises of 105 parts, 34 of which were supplied by the market; the rest were manufactured in-house. **Figure E-3.**
E-2.3 Electrical and Electronic Design

The electrical and electronic design was largely based on the previous one, yet it was improved in several aspects, Figure E-4. Power autonomy was achieved using Li-Po batteries. Two DC/DC converters were also employed to supply two optionally isolated circuits (otherwise a common ground connects them): a low-voltage circuit for the computer, the sensors and the wireless modules (nominal voltage 5V), and a high-voltage circuit for the servomotors and the thruster valves (nominal voltage 24V).

Figure E-4. Electrical and electronics subsystem and its interconnections: red lines represent power and blue lines signals.

PCBs have been designed and printed for power routing, thrusters’ control, collecting optical sensors’ data (this PCB is called “Arduino shield”), and a dedicated PCB has been developed to be used as a control panel, Figure E-5a. In order to attain computation autonomy a PC-104 system is...
used, including a CPU board (running at 500 MHz), a 48 Digital I/O card and USB flash drive (for booting), Figure E-5b.

![Figure E-5](image)

(a) Close up of PCBs and (b) PC104 board.

The localization of the robot is achieved through two different sensors: (a) relative estimation sensors, Figure E-6a, which are comprised by three optical sensors mounted at the bottom of the chassis, for fast estimation and (b) an absolute estimation sensor, that is an off-board overhead camera, which locates LEDs on top of the robot Figure E-6b, providing more accurate estimation. An Arduino board with a custom shield is used for collecting optical sensor data. An off-board computer receives data from the camera, does the image processing (in C/C++) and finally transmits the robot pose via UDP/IP (faster than TCP/IP, used in the past) to the on-board CPU. Two wireless Ethernet bridges are used (a) for the remote control of the robot, and (b) to receive real-time data wirelessly from the external camera system, Figure E-7.

![Figure E-6](image)

(a) Optical Sensors and (b) LEDs on top of the robot.
E-2.4 Software Design through Model-Based Design

A modern aspect considers the traditional, text-based approach of embedded system design inefficient to handle advanced control systems development, and indicates that the divide-and-conquer strategy for developing such systems, which means coordinating the resources of people with expertise in a wide range of disciplines, leads to slow development cycles. Focusing also on the experience gained from the first robot’s design process, it was clear that developing complex models was difficult, time-consuming, and highly prone to errors. In addition, debugging text-based programs for the first robot was, and still is, a tedious process, requiring much trial and error before a final fault-free model could be created, especially since mathematical models undergo unseen changes during the translation through the various design stages. Trying to eliminate these time-consuming problems underlying the traditional design process, a different approach, called Model-Based Design (MBD), was tried out for the new robot.

Hence, employing this technique, the development of the robot was carried out as a whole, since the design of algorithms, the software development and the final integration on the hardware were studied as interdependent parts. Development was manifested in four steps: (1) modeling the plant, (2) synthesizing a controller for the plant, (3) simulating the plant and the controller, and (4) deploying the controller on real hardware after connecting it to sensors and actuators of the actual plant. An iterative debugging process was carried out by analyzing results on the actual target and updating the controller model. Model based design tools allowed these iterative steps to be performed in a unified visual environment.

The software used was the xPC Target, from Mathworks. In this environment a Matlab/Simulink model (running on a host PC) is auto-generated into C code, compiled with a third-party
compiler, transmitted wirelessly to the robot (target PC) and finally runs in a real-time kernel, in
Figure E-8. Remote control, monitoring, parameter tuning as well as virtual reality animation
(with VRML) were also enabled and extensively used during the development. In this manner,
validation and verification of the design were continuously performed throughout the development.
As a result, emphasis was put on innovation, while at the same time potential low-level problems
in hardware and software were surmounted.

However, during the development phase, several compatibility issues, regarding xPC Target,
came to surface, which complicated the design process and finally imposed strict limitations for the
hardware selection. Generally, two major problems had to be addressed. The first one concerned
the fact that the commonly used USB connection between the optical sensors and the robot CPU
could not be implemented within the xPC Target software, since it is a disabled feature by default.
To overcome this inconvenience, an Arduino microcontroller was interposed to establish the
respective communication, employing the PS/2 protocol to collect optical sensor data, and the RS-
232 protocol to transmit them (with or without post-processing) to the CPU. To this end, an
Arduino optical sensors shield was designed and manufactured.

A second problem concerned the limited reference for wireless communications within the
xPC Target. The solution given was to use two wireless Ethernet bridges connected to the CPU
Ethernet ports. This configuration defined the CPU board selection among a very limited catalog of
supported PC-104 boards which complied with the requirements. Special focus was finally placed
on the real-time wireless communication between the robot’s PC (target PC) and the camera PC.
This communication required a dedicated link (a second wireless bridge) and a software
conversion from TCP (previously used) to the UDP/IP communication protocol.

![Figure E-8. Interconnections between PC104, subsystems and external computers using xPC Target](image-url)
E-2.5 Robotics Operating System (ROS) Implementation

CSL Space Emulator can use various OS (windows, linux, etc). However the advantages of Robotic Operating System (ROS) were acknowledged and currently all robotic systems as well as their communications with the workstations use ROS, Figure E-10. The implementation is at the last phases, and the currently the localization subsystems and their programming are integrated into ROS.

![Figure E-9. Instants of ROS environment in CSL.](image)

In a glance the current implementation of the emulator has:

**Architecture:** (a) Local and Global localization for better support of different docking phases and (b) Distributed system with synced Kernel clock’s for usable data stamping

**Hardware:** (a) PC104 form factor for modular design, (b) CPU Intel Atom 1.8GHz for optimal performance to power consumption ratio, (c) Wireless connectivity via Wifi, and (d) TCP packets to ensure reliable data handling.

**Software:** (a) Distributed code for simplicity and better use of dual-core system, (b) Hardware specific software deployment to minimize CPU bandwidth, and (c) Fail safe design.

E-2.6 Future Developments

The servomotors subsystem is yet incomplete; however there have been preliminary selections for the components of this subsystem. There will be at least five motors with servomotors on board: Two sets of two motors/ servomotors for controlling two 2-DOF manipulators and one motor/ servomotor for a reaction wheel. Due to the modularity of the design, the motors can be used also for controlling specialized payloads if necessary, instead of the manipulators. Note that at the current configuration the new robot can easily emulate a free-floating satellite with or without initial motions imposed by the use of the thrusters.
E-3 Validation Experiments

To validate the functionality of the new robot and most specifically the combination of the localization subsystems and the xPC Target, a number of experiments have taken place. The robot was forced to move through circular or straight paths. The motion as captured by the overhead camera is presented in Figure E-10 and Figure E-11. Similarly, a circular motion as seen by the optical sensors is presented in Figure E-12. However, these experiments only show a rough verification of subsystem functionality, since the sensors have not been calibrated yet. Calibration will be achieved by employing a Phasespace mocap system.

![Figure E-10](image1.png)

**Figure E-10.** Five experiments on circular trajectories with data received from the camera system. (White: desired trajectory, Red: data after image processing).

![Figure E-11](image2.png)

**Figure E-11.** Five experiments on straight line trajectories with data received from the camera system.
Figure E-12. Motion data derived from the optical sensors system during a circular trajectory experiment.

E-4 Full Emulator

Finally both robots have been positioned on the granite table, Figure E-13. Their coexistence on the table was very smooth and allows for interesting experiments in the near future.

Figure E-13. NTUA Space Robot Emulator with both robots.